The 6th Workshop on Recent Development of Mathematical Fluid Dynamics and Hyperbolic Conservation Laws

August 28–30, 2026

Program & Abstract

Nagoya Institute of Technology, Nagoya, Japan

Date

August 28–30, 2026

Venue

Lecture Room 5234, 3rd Floor, Building 52, Nagoya Institute of Technology, Japan

Organizers

Chun-Hsiung Hsia (National Taiwan Univ.)
Bongsuk Kwon (UNIST)
Masahiro Suzuki (Nagoya Inst. of Tech.)
Yoshihiro Ueda (Kobe Univ.)

Sponsors

JSPS Grant-in-Aid for Scientific Research (B) 25K00915 (M.Suzuki)

Program

Friday, August 28

10：00〜12：00	Masashi Ohnawa (Tokyo Univ. of Marine Sci. and Tech.) Stability of sub/trans/super-critical flows for some fluid models
14：00〜14：50	In-Jee Jeong (KIAS) Stability of multiple Lamb dipoles
15：00〜15：50	Kazuyuki Tsuda (Kyushu Sangyo Univ.) Time periodic problem of the Navier-Stokes equations in an exterior domain with periodically moving boundary
16：10〜17：00	Jaeyong Shin (Yonsei University) Some global solutions to the electron and Hall MHD equations

Saturday, August 29

10：00〜12：00	Moon-Jin Kang (KAIST) The a-contraction method for stability of shocks
14：00〜14：50	I-Kun Chen (National Taiwan Univ.) On thermal transpiration and thermomolecular pressure difference
15：00〜15：50	Shota Sakamoto (Kyushu Univ.) Uniqueness of weak solutions to the non cut-off Boltzmann equation
16：10〜17：00	Kung Chien Wu (National Cheng Kung Univ.) Time-asymptotic expansion and fluid approximation of the 1D Boltzmann equation with hard potentials

Sunday, August 30

10：00〜12：00	Jin-Cheng Jiang (National Tsing Hua Univ.) Derivation of Fokker-Planck-Landau equation from Boltzmann equation by weak-coupling limit
14：00〜14：50	Kai Koike (Keio Univ.) Relaxation enhancement by controlled Euler flow
15：00〜15：50	Cheng-Fang Su (National Yang Ming Chiao Tung Univ.) Strong Solutions and Blow-up Criteria for the Compressible Navier–Stokes Equations with Inflow–Outflow Boundary Conditions
16：10〜17：00	Wanyong Shim (KAIST) Asymptotic self-similar blow-up for the regularized Saint–Venant equations

Stability of sub/trans/super-critical flows for some fluid models

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There are some PDE's which admit stationary solutions or traveling waves that change those profiles qualitatively depending on their parameters like boundary conditions or far field asymptotic values. In this lecture, I consider traveling wave solutions to simplified model equation for gas dynamics with radiation

$$\begin{aligned} u_t + uu_x + q_x &= 0, \\ -q_{xx} + q + u_x &= 0 \end{aligned}$$

in $\mathbb{R}$, and gas flows in a tube with variable cross section

$$\begin{aligned} (A\rho)_t + (A\rho u)_x &= 0, \\ (A\rho u)_t + (A(\rho u^2 + p))_x &= A_x p \end{aligned}$$

in a bounded interval as examples, and will show different types of profiles have different mechanisms for stabilization.

Stability of multiple Lamb dipoles

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Lamb dipole is an explicit traveling wave solution of the two-dimensional incompressible Euler equations, described by Lamb back in 1895 ([2]). Due to difficulties arising from the fact that it is the dipole with "maximal mass" under enstrophy and impulse constraints, its nonlinear orbital stability was proved only in 2022 by Abe and Choi ([1]). Our main result is the nonlinear orbital stability of linear superpositions of Lamb dipoles, allowing for dipoles with mixed signs. Such a configuration is not a local extremizer of the kinetic energy, which is the main challenge in applying the variational principle to obtain stability. Furthermore, when dipoles have mixed signs, the impulse is not coercive anymore. These issues are handled by Lagrangian bootstrapping schemes, which carefully track the space-time location of various parts of the solution. This is based on joint works with Ken Abe, Kyudong Choi, Guolin Qin, and Yao Yao.

References

[1] K. Abe and K. Choi. Stability of Lamb dipoles. *Arch. Ration. Mech. Anal.*, 244(3):877–917, 2022.

[2] H. Lamb. *Hydrodynamics.* Cambridge Univ. Press., 2nd edition, 1895.

Time periodic problem of the Navier-Stokes equations in an exterior domain with periodically moving boundary

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We consider the time periodic problem of the Navier-Stokes equations on a non-cylindrical space-time domain. For $t \in \mathbb{R}$ let $\Omega(t) \subset \mathbb{R}^n$, $n \geq 3$, be a family of exterior domains considered as perturbation of an exterior reference domain $\Omega_0 \subset \mathbb{R}^n$ with $\partial\Omega(t) \in C^3$, and set $Q := \bigcup_{t\in\mathbb{R}} \Omega(t) \times \{t\}$. We assume that $\Omega(t)$ is time periodic with period $T > 0$, *i.e.* $\Omega(t+T) = \Omega(t)$, and that for simplicity $\Omega(0) = \Omega_0$. Suppose that $\overline{\Omega(t)^c}$ is contained in an open ball B_R of radius $R > 0$ for all times t. The Navier-Stokes equations on the non-cylindrical space-time domain Q are described by

$$\begin{aligned} v_t - \nu\Delta v + v \cdot \nabla v + \nabla p = f \quad &\text{in} \quad Q, \\ \operatorname{div} v = 0 \quad &\text{in} \quad Q, \\ v = 0 \quad &\text{on} \quad \bigcup_{t\in\mathbb{R}} \partial\Omega(t) \times \{t\}. \end{aligned}$$

Here v denotes the unknown velocity of an incompressible, viscous fluid and p denotes the pressure, respectively, at time $t \in \mathbb{R}$ and position $x \in \Omega(t)$. Moreover, let $f = f(x,t)$ be a given external force satisfying $f(x, t+T) = f(x,t)$. For simplicity, assume that $v = 0$ on $\partial\Omega(t)$; the correct kinematic boundary condition will lead by classical trace arguments to a Navier-Stokes system with perturbation terms which are of order zero and one in v. Moreover the coefficient of viscosity ν is equal to 1.

For this case we prove the existence of a locally unique mild time periodic solution in weighted function spaces with radially symmetric Muckenhoupt weights. This type of problem was classically studied by Morimoto (1971) and Miyakawa and Teramoto (1982), after global weak solutions was shown by Fujita and Sauer (1968), and has subsequently been investigated by several authors. We briefly review previous results on fixed or periodically moving boundaries.

To solve our problem, a Muckenhoupt weight approach works well. Indeed, the solutions split into a stationary part controlled by potential theoretic estimates and a purely oscillatory part constructed as mild solution via analytic semigroup theory in weighted spaces. To deal with perturbation terms of even second order—coming from a coordinate transform and the moving boundary—a maximal L^1 type regularity estimate will be used in weighted Lorentz spaces. To control the convective term an H^∞-calculus in weighted spaces of the Stokes operator, its BIP property and embedding estimates of fractional powers are exploited.

Some global solutions to the electron and Hall MHD equations

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In this talk, we discuss several classes of global solutions to the resistive electron MHD and Hall MHD equations. We first study the electron MHD equations in two-dimensional and axisymmetric settings. In each setting, the equations admit a propagated structure, which we call the normal structure. Under this structure, the equations reduce to a scalar parabolic equation and admits global solutions that are smooth for positive times.

A main example arises from the axisymmetric electron MHD equations with purely azimuthal magnetic field $B = b^\theta e_\theta$. In this case, $\Pi = b^\theta / r$ satisfies a viscous Burgers-type equation, and Nash iteration yields global well-posedness and decay estimates. We then discuss stability around these normal structures and construct global solutions to the electron MHD equations. We also explain how these ideas extend to the Hall MHD system. If time permits, we present a special class of Hall MHD solutions known as double Beltrami states. This talk is based on recent and ongoing joint works with Hantaek Bae (UNIST) and Kyungkeun Kang (Yonsei University).

The a-contraction method for stability of shocks

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In this lecture, I will give a brief introduction to the method of a-contraction with shifts (in short, a-contraction method), highlighting its main ideas, which has been developed over the past decade for the stability analysis of viscous shock waves arising in viscous compressible fluid equations, especially for the Navier-Stokes equations (without or with boundary). Based on this framework, I will present the results on the long-time dynamics for shock waves or generic superposition containing shocks. If time permits, I will also briefly discuss the recent work on the hydrodynamic limit of the Boltzmann equation towards shocks.

On thermal transpiration and thermomolecular pressure difference

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We demonstrate the phenomenon of thermal transpiration in a bounded convex domain. We employ the stationary Boltzmann equation with a cutoff potential. For boundary condition, we partition the boundary into diffuse reflection and incoming regions. We establish the existence of solution in a weighted L^∞ space. Furthermore, we consider a convex domain with diffuse reflection boundary condition in the middle and incoming boundary condition at the two ends. We first consider Maxwellians with the same pressure but different temperatures at the two ends, $T_1 = 1$ and $T_2 > 1$. We prove that the total flux $U(x)$ is directed toward the hot end. Furthermore, we derive an estimate for the total flux:

$$U(x) > C(1 - \frac{1}{\sqrt{T_2}})$$

In addition, we show that when the pressures and temperatures on the two ends satisfy the relation

$$\frac{P_1}{P_2} = \frac{\sqrt{T_1}}{\sqrt{T_2}}$$

the total flux of the solution is of order $O(\frac{1}{k})$. This result is consistent with Knudsen's finding of thermomolecular pressure difference in 1909.

Uniqueness of weak solutions to the non cut-off Boltzmann equation

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Joint work with Dingqun Deng (Akita University)

The uniqueness of solutions to the non cut-off Boltzmann equation

$$\partial_t F(t,x,v) + v\cdot\nabla_x F(t,x,v) = Q(F,F)(t,x,v) \quad \text{in } (0,T)\times\mathbb{R}^d_x\times\mathbb{R}^d_v,$$

where $Q(F,G)(v) = \int_{\mathbb{R}^d_{v_*}}\int_{\mathbb{S}^{d-1}_\sigma} B(v-v_*,\sigma)\big(F(v'_*)G(v')-F(v_*)G(v)\big)\,d\sigma dv_*$ is the collision operator, $v' = \frac{v+v_*}{2}+\frac{|v-v_*|\sigma}{2}$, $v'_* = \frac{v+v_*}{2}-\frac{|v-v_*|\sigma}{2}$ $(\sigma\in\mathbb{S}^{d-1})$, $B(v-v_*,\sigma) = |v-v_*|^\gamma b(\cos\theta)$, $\theta^{-1-2s}\le \sin^{d-2}\theta\, b(\cos\theta)\simeq\theta^{-1-2s}$, has been a well-known open problem in the community. There are some known literature for uniqueness under smallness and/or regularity assumptions of initial data, however, uniqueness for large and non-smooth data is mostly unknown. Concerning this problem, the following theorem recently obtained will be presented in this talk: Let $\mu(v) = (2\pi)^{-\frac{d}{2}}e^{-\frac{|v|^2}{2}}$ be the normalized Maxwellian and substitute $F=\mu+\mu^{\frac12}f$. Then the Cauchy problem for f reads

$$\begin{cases}\partial_t f + v\cdot\nabla_x f = \Gamma(\mu^{\frac12}+f,f)+\Gamma(f,\mu^{\frac12}), & \text{in } (0,T)\times\mathbb{R}^d_x\times\mathbb{R}^d_v,\\ f(0,x,v)=f_0, & \text{in } \mathbb{R}^d_x\times\mathbb{R}^d_v.\end{cases} \tag{1}$$

Theorem 1 (Uniqueness and $L^2_{t,x,v}$ stability [1])**.** *Let $d\ge 2$ and fix any $M_0, T_*>0$. Assume γ and s satisfy $\gamma+2s<0$ and $\gamma+s>-\frac{d}{2}$. There exist sufficiently large constants $r=r(d,\gamma,s)\in(2,\infty)$ and $C_{\gamma,s,d}>0$ such that, if ϕ_1 and ϕ_2 are two solutions to the non-cutoff Boltzmann equation (1) on $[0,T_*]$ with initial data $\phi_{1,0}$ and $\phi_{2,0}$, respectively, and they satisfy*

$$\Phi_1(t,x,v) := \mu+\mu^{\frac12}\phi_1 \ge C_{\mathrm{low}}^{-1}\mu^{L_0}, \quad a.e.\ (t,x,v)\in[0,T]\times\mathbb{R}^{2d}_{x,v}, \tag{2}$$

$$\begin{aligned}&\|\langle v\rangle^{C_{\gamma,s,d}}\phi_2\|_{L^\infty_t([0,T_*])L^r_{x,v}} + \|\langle v\rangle^{C_{\gamma,s,d}}\phi_2\|_{L^\infty_t([0,T_*])L^2_{x,v}} \le M_0,\\ &\sum_{j\ge 0}(j+1)^2\|\Delta_j\langle v\rangle^{|\gamma|+d+1}\phi_1\|^2_{L^\infty_t([0,T])L^\infty_x L^r_v}\le M_0,\end{aligned} \tag{3}$$

where $\{\Delta_j\}$ is the inhomogeneous Littlewood-Paley decomposition and $C_{\mathrm{low}}, L_0>0$ are fixed constants, then we have the $L^2_{t,x,v}$ stability estimate

$$\|\phi_1-\phi_2\|_{L^2_t([0,T_*])L^2_{x,v}} \le C_{T_*,M_0,\gamma,s,d,C_{\mathrm{low}},L_0}\|\phi_{1,0}-\phi_{2,0}\|_{L^2_{x,v}}.$$

Consequently, if $\phi_{1,0}=\phi_{2,0}$, we have the following uniqueness:

$$\phi_1(t,x,v)=\phi_2(t,x,v) \quad \textit{for a.e.} \quad (t,x,v)\in[0,T]\times\mathbb{R}^d_x\times\mathbb{R}^d_v.$$

References

[1] D. Deng and S. Sakamoto, preprint. `https://arxiv.org/abs/2602.15601`

Time-asymptotic expansion and fluid approximation of the 1D Boltzmann equation with hard potentials

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We investigate the quantitative pointwise behavior of solutions to the one-dimensional (1D) Boltzmann equation for hard potentials and Maxwellian molecules. Our study pursues two main objectives. First, we characterize the time-asymptotic hierarchical structure of solutions. Second, we establish the asymptotic equivalence between solutions of the 1D Boltzmann equation and the 1D compressible Navier-Stokes equations. Unlike prior works in the multi- dimensional case, where comparison of linear solutions suffices, our approach extracts the leading nonlinear fluid waves from the 1D Boltzmann equation and directly compares them with solutions of the nonlinear Navier-Stokes system.

Derivation of Fokker-Planck-Landau equation from Boltzmann equation by weak-coupling limit

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The standard physics literature establishes that for any smooth (and sufficiently decaying at infinity) potential function $\Phi(|x|)$, the Boltzmann equation converges to the Fokker-Planck-Landau equation in the weak coupling limit, with the characteristic diffusion coefficient given by

$$M_\phi := \int_0^\infty |\hat{\Phi}(r)|^2 r^3 dr.$$

We present a rigorous mathematical justification of this physical principle through two key contributions: (1) We develop a new scaling for the collision kernel of the Boltzmann operator, which provides a mathematically well-defined framework for the weak coupling limit. This scaling precisely captures the dynamics of weak coupling interactions, including those governed by inverse power law or Coulomb potentials. (2)Within a perturbation theory framework, we conduct a thorough mathematical analysis of the global asymptotic behavior in the weak coupling regime. This talk is based on the joint work with Laurent Desvillettes, Ling-Bing He and Yu-long Zhou.

Relaxation enhancement by controlled Euler flow

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Vahagn Nersesyan, New York University Shanghai
Manuel Rissel, ShanghaiTech University
Marius Tucsnak, Université de Bordeaux

Consider the transport–diffusion equation

$$\begin{cases} \partial_t \phi + v \cdot \nabla \phi - \Delta \phi = 0 & (t \geqslant 0, x \in \mathbb{T}^2), \\ \phi(0,x) = f(x) & (x \in \mathbb{T}^2), \end{cases} \tag{1}$$

where ϕ is a scalar unknown, v is a divergence-free vector field, and f is initial data. Following [CKRZ08], we define the concept of relaxation enhancing vector field.

Definition 1. A stationary divergence-free vector field $\tilde{v}$ on $\mathbb{T}^2$ is *relaxation enhancing* if for every $\tau, \delta > 0$, there exists $\bar{A} > 0$ such that for any $A \geq \bar{A}$ and every $f \in L^2(\mathbb{T}^2)$ with $\|f\|_{L^2(\mathbb{T}^2)} \leq 1$ and $\int_{\mathbb{T}^2} f(x)\,dx = 0$, the solution ϕ to (1) with $v = A\tilde{v}$ satisfies

$$\|\phi(\tau,\cdot)\|_{L^2(\mathbb{T}^2)} < \delta.$$

A spectral characterization of relaxation enhancing vector fields and their examples are given in [CKRZ08]. Our question is whether we can construct (a time-dependent variation of) relaxation enhancing vector fields as solutions to controlled Euler equations:

$$\begin{cases} \partial_t v + (v \cdot \nabla) v + \nabla p = \displaystyle\sum_{j=1}^{m} u_j(t)\theta_j(x) & \text{in } (0,\infty) \times \mathbb{T}^2, \\ \operatorname{div} v = 0 & \text{in } (0,\infty) \times \mathbb{T}^2, \\ v(0,\cdot) = v_0 & \text{on } \mathbb{T}^2. \end{cases} \tag{2}$$

Here, $(\theta_j)_{j=1}^m$ are fixed solenoidal vector fields and $(u_j)_{j=1}^m$ are *controls* we construct so that the solution v to (2) is relaxation enhancing in the following sense:

Theorem 2 ([KNRT25])**.** *Let $m = 4$ and*

$$\begin{aligned} \theta_1(x) &= {}^T(0,1)\sin(x_1), & \theta_2(x) &= {}^T(0,1)\cos(x_1), \\ \theta_3(x) &= {}^T(1,1)\sin(x_1 - x_2), & \theta_4(x) &= {}^T(1,1)\cos(x_1 - x_2). \end{aligned}$$

For any $v_0 \in H^4_\sigma(\mathbb{T}^2)$ and $\tau, \delta > 0$, there exist $(u_j)_{j=1}^4 \subset C^\infty([0,\tau])$ such that the solution ϕ to (1) where v is the solution to (2) satisfies

$$\|\phi(\tau,\cdot)\|_{L^2(\mathbb{T}^2)} < \delta$$

for all $f \in L^2(\mathbb{T}^2)$ with $\|f\|_{L^2(\mathbb{T}^2)} \leq 1$ and $\int_{\mathbb{T}^2} f(x)\,dx = 0$.

References

[CKRZ08] P. Constantin, A. Kiselev, L. Ryzhik, and A. Zlatoš. Diffusion and mixing in fluid flow. *Ann. of Math. (2)*, 168(2):643–674, 2008.

[KNRT25] K. Koike, V. Nersesyan, M. Rissel, and M. Tucsnak. Relaxation enhancement by controlling incompressible fluid flows. *arXiv:2506.22233*, 2025.

Strong Solutions and Blow-up Criteria for the Compressible Navier–Stokes Equations with Inflow–Outflow Boundary Conditions

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In this talk, we discuss the strong-solution theory for a compressible Navier–Stokes–Fourier type system in a bounded domain under inflow–outflow boundary conditions. In contrast with impermeable boundary conditions, the inflow boundary requires prescribed density data, and differentiating the continuity equation produces boundary terms that cannot be eliminated by the usual cancellation mechanisms. These boundary contributions constitute one of the main analytical difficulties of the problem and require a boundary-sensitive energy method. We present a local-in-time existence result for strong solutions with nonhomogeneous velocity and temperature boundary data, and then discuss a continuation criterion, equivalently a blow-up criterion, for such solutions. Particular emphasis will be placed on how the inflow boundary affects the density estimates and why this boundary structure necessitates a modification of the standard energy method. The talk concludes with an overview of how these estimates yield the continuation of strong solutions and the corresponding blow-up criterion. This is joint work with Professor Ching-Hsiao Cheng and Young-Sam Kwon.

Asymptotic self-similar blow-up for the regularized Saint–Venant equations

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We study singularity formation in the regularized Saint–Venant (rSV) equations in one space dimension, formally regarded as a Hamiltonian regularization of the isentropic Euler equations. Although smooth solutions to the rSV system are known to develop finite-time gradient blow-up, the precise structure of such singularities has not been rigorously characterized. In this talk, we present a sharp description of the asymptotically self-similar blow-up profile and establish its $C^{3/5}$ Hölder regularity at the singular time. This regularity differs from the $C^{1/3}$ singularity observed in the isentropic Euler equations. This contrast motivates a discussion of the structural influence of the Hamiltonian regularization on singularity formation.